

**ÉRETTSÉGI VIZSGA • 2022. május 3.**

# MATEMATIKA ANGOL NYELVEN

## EMELT SZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

## JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

EMBERI ERŐFORRÁSOK MINISZTERIUMA

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## Instructions to examiners

### Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
  - correct calculation: *checkmark*
  - principal error: *double underline*
  - calculation error or other, not principal, error: *single underline*
  - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
  - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
  - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

### Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

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6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
  7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
  8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
  9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
  10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**:  
addition, subtraction, multiplication, division, calculating powers and roots,  $n!$ ,  $\binom{n}{k}$ ,  
replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers  $\pi$  and  $e$ , finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
  11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
  12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
  13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
  14. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

**I.**

<b>1. a) Solution 1</b>		
The smallest possible sum is 7 so, besides the 1-s, either a single 3 or two 2-s must be rolled.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
There are 7 different ways to roll six 1-s and a single 3.	1 point	
There are $\binom{7}{2} (= 21)$ different ways to roll five 1-s and two 2-s.	1 point	$\frac{7!}{2! \cdot 5!}$
There are a total $7 + 21 = 28$ different options.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>1. a) Solution 2</b>		
The smallest possible sum is 7, the remaining 2 must be “shared” between the 7 rolls.	1 point	
Select two out of the seven rolls, without order, with possible repeat. This is a case of class 2 combinations of 7 items with repeat.	1 point	
There are $\binom{7+2-1}{2} = 28$ such cases.	2 points	
<b>Total:</b>	<b>4 points</b>	

*Note: Award maximum score if the candidate gives the correct answer using a logically ordered list of all possibilities.*

<b>1. b)</b>		
The data arranged in increasing order: 1, 2, 3, 3, 4, 5, 5. If a 1 or 2 or 3 is rolled the median will be 3. If a 4 or 5 or 6 is rolled the median will be 3.5.	2 points	
The sum of the first 7 numbers is 23. If a 1, 2, ... 6 is rolled, the mean will be $\frac{24}{8} (= 3), \frac{25}{8}, \frac{26}{8}, \frac{27}{8}, \frac{28}{8} (= 3.5), \frac{29}{8}$ , respectively.	2 points	
The mean will be greater than the median in case a 2, 3 or 6 is rolled.	2 points	
<b>Total:</b>	<b>6 points</b>	

<b>1. c) Solution 1</b>																																																			
The total number of cases is $6 \cdot 6 = 36$ .	1 point	<table border="1"> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr> <tr><td>1</td><td></td><td>×</td><td>×</td><td>×</td><td>×</td><td>×</td></tr> <tr><td>2</td><td></td><td></td><td>×</td><td>×</td><td>×</td><td>×</td></tr> <tr><td>3</td><td></td><td></td><td></td><td>×</td><td>×</td><td>×</td></tr> <tr><td>4</td><td></td><td></td><td></td><td></td><td>×</td><td>×</td></tr> <tr><td>5</td><td></td><td></td><td></td><td></td><td></td><td>×</td></tr> <tr><td>6</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>		1	2	3	4	5	6	1		×	×	×	×	×	2			×	×	×	×	3				×	×	×	4					×	×	5						×	6						
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If the first number is a 1, there will be 5 different options for the second (2, 3, 4, 5, 6). If the first number is a 2, there will be 4 different options, if 3, there will be 3, if 4, there will be 2, if 5, there will be 1 (if 6, there will be none). The number of favourable cases is $(5 + 4 + 3 + 2 + 1 =) 15$ .	2 points																																																		
The probability is $\frac{15}{36}$ ( $\approx 0.417$ ).	1 point																																																		
<b>Total:</b>	<b>4 points</b>																																																		

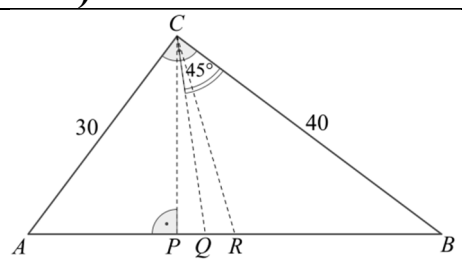
<b>1. c) Solution 2</b>		
The probability that the two numbers rolled are the same is $\frac{1}{6}$ .	1 point	
The probability of either number being greater than the other is the same for both.	1 point	
The probability that the second number will be greater is $1 - \frac{1}{6} = \frac{5}{6}$ ( $\approx 0.833$ ).	2 points	
<b>Total:</b>	<b>4 points</b>	

<b>2. a)</b>		
(1) false (2) true (3) false (4) false	3 points	<i>Award 2 points for three correct answers, 1 point for two correct answers and 0 points for less than two correct answers.</i>
<b>Total:</b>	<b>3 points</b>	

<b>2. b)</b>		
$(x + 2)^2 + (y - 3)^2 = 13 - c$	2 points	
If $c \leq 12$ then $13 - c > 0$ and so this is the equation of a circle (of centre $(-2; 3)$ and radius $\sqrt{13 - c}$ ).	1 point	
The statement is true.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>2. c)</b>		
The converse of the statement: If the equation $x^2 + 4x + y^2 - 6y + c = 0$ is that of a circle then $c \leq 12$ .	1 point	<i>Equivalent statements may also be accepted.</i>
A counterexample is $c = 12.75$ (in this case the radius of the circle is $\sqrt{13 - c} = 0.5$ ).	1 point	<i>Counterexample: <math>12 &lt; c &lt; 13</math>.</i>
The converse of the statement is false.	1 point	
<b>Total:</b>	<b>3 points</b>	

<b>3. a)</b>		
Let $q$ be the common ratio of the geometric sequence, and $x$ be the length of the third side in cm-s. (There will be three cases in terms of the length $x$ .) If $x < 12$ , then $\frac{x}{12} = \frac{12}{27}$ , so $x = \frac{16}{3}$ ( $\approx 5.33$ ).	1 point	$x < 12$ is not possible as $x + 12 < 27$ would make an impossible triangle.
If $x > 27$ , then $\frac{x}{27} = \frac{27}{12}$ , so $x = \frac{243}{4}$ ( $= 60.75$ ).	1 point	If $x > 27$ then $q > 2$ and so the third side would be greater than $2 \cdot 27 = 54$ .
These two cases will not provide a triangle as they violate the triangle-inequality.	1 point	$12 + 27 < 54$ so this triangle does not exist.
If $12 < x < 27$ , then $\frac{x}{12} = \frac{27}{x}$ , so $x = \sqrt{12 \cdot 27} = 18$ ( $x > 0$ ).	1 point	
$x = 18$ cm is correct (as $12 + 18 > 27$ ).	1 point	
<b>Total:</b>	<b>5 points</b>	

<b>3. b)</b>		
 <p>(Let's first determine the length of segments <math>AB</math>, <math>AP</math>, <math>AQ</math> and <math>AR</math>.) According to the Pythagorean theorem <math>AB = 50</math>. Hence <math>AR = RB (= AB : 2) = 25</math>.</p>	1 point	
According to the Leg (geometric mean) theorem $AP \cdot AB = AC^2$ , so $AP = 900 : 50 = 18$ .	2 points	$BP = 32$
According to the Angle bisector theorem: $AQ : QB = 3 : 4$ , so $AQ = \frac{3}{7} \cdot AB = \frac{150}{7}$ .	2 points	$BQ = \frac{200}{7}$
$PQ = (AQ - AP = \frac{150}{7} - 18 =) \frac{24}{7}$ $QR = (AR - AQ = 25 - \frac{150}{7} =) \frac{25}{7}$	1 point	
$AP : PQ : QR : RB = 18 : \frac{24}{7} : \frac{25}{7} : 25 = 126 : 24 : 25 : 175$	1 point	
<b>Total:</b>	<b>8 points</b>	

Note: Deduce 1 point if the candidate uses any approximate values.

<b>4. a)</b>		
The flight time for the plane is $\frac{1200}{750} = 1.6$ hours.	1 point	
During this time the aircraft burns $1.6 \cdot 2.4 = 3.84$ tons of jet fuel	1 point	
which costs $3.84 \cdot 900 = 3456$ euros.	1 point	
This covers the transportation of 150 passengers to a distance of 1200 km. To transport one person to a distance of 1 km costs about $\frac{3456}{150 \cdot 1200} = 0.0192$ euros.	1 point	
The passenger car consumes 6 litres of fuel on 100 km which costs $6 \cdot 1.2 = 7.2$ euros.	1 point	
This covers the transportation of 5 people to a distance of 100 km. To transport 1 person to a distance of 1 km costs $\frac{7.2}{5 \cdot 100} = 0.0144$ euros.	1 point	
Based on the fuel costs only, the car is more economical.	1 point	
<b>Total:</b>	<b>7 points</b>	

<b>4. b)</b>		
Let $x$ be the number of menus sold.		
The number of sandwiches sold separately is then $\frac{x}{2}$ the number of soft drinks sold separately is $x + 10$ .	2 points	
The revenue from sales outside of menus is $\frac{x}{2} \cdot 3.5 + (x + 10) \cdot 3 + 28 \cdot 2.5 = 4.75x + 100$ (euros).	1 point	
The revenue from menu sales is $5.5x$ , so $2 \cdot 5.5x = 4.75x + 100$ .	1 point	
Here $6.25x = 100$ , that is $x = 16$ .	1 point	
The total revenue (16 menus, 8 more sandwiches, 26 soft drinks and 28 coffees) is $16 \cdot 5.5 + 8 \cdot 3.5 + 26 \cdot 3 + 28 \cdot 2.5 = 264$ euros.	1 point	
Check: Revenue from menu sales is $5.5 \cdot 16 = 88$ euros, which is exactly one third of the total revenue.	1 point	
<b>Total:</b>	<b>7 points</b>	

## II.

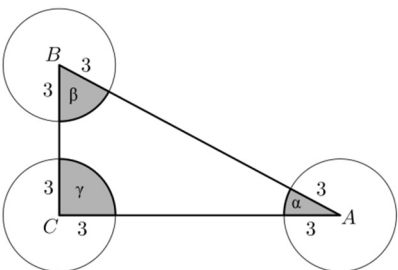
<b>5. a)</b>		
The greatest angle of the triangle is opposite the side $a + 2$ .	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
Apply the Law of Cosines for angle $\gamma$ : $\cos \gamma = \frac{a^2 + (a+1)^2 - (a+2)^2}{2a(a+1)} =$	1 point	
$= \frac{a^2 - 2a - 3}{2a(a+1)} =$	1 point	
$= \frac{(a+1)(a-3)}{2a(a+1)} =$	2 points*	
$= \frac{a-3}{2a} \quad (a \neq -1).$	1 point*	
<b>Total:</b>	<b>6 points</b>	

Note: The 3 points marked by \* may also be given for the following reasoning:

We are proving that $\frac{a^2 - 2a - 3}{2a(a+1)} = \frac{a-3}{2a}$ .	1 point	
Multiply by the (positive) denominator of the fraction on the left: $a^2 - 2a - 3 = (a-3)(a+1)$ .	1 point	
This is an identity. As all steps were equivalent, the statement is proven.	1 point	

<b>5. b)</b>		
$\cos 120^\circ = -\frac{1}{2} = \frac{a-3}{2a}$ ,	1 point	
$a = 1.5$ .	1 point	
The three sides of the triangle are 1.5, 2.5 and 3.5 units (and such triangle truly exists).	1 point	
<b>Total:</b>	<b>3 points</b>	



<b>5. c)</b>		
 <p>Use the symbols of the diagram.          (All three sides are longer than 6 cm and so) the points that are at least 3 cm apart from each vertex are in the white part of triangle <math>ABC</math> (and also on the edges of this region). The probability is the ratio of this area against the full area of the triangle.</p>	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
The area of the right triangle is $\frac{8 \cdot 15}{2} = 60$ (cm <sup>2</sup> ).	1 point	
$\alpha + \beta + \gamma = 180^\circ$ , so the combined area of these three sectors is exactly half of the area of a circle of radius 3 cm,	2 points*	
which is $4.5\pi \approx 14.14$ (cm <sup>2</sup> ).	1 point*	
The area of the white region is therefore $(60 - 4.5\pi \approx) 45.86$ (cm <sup>2</sup> ).	1 point	$1 - \frac{4.5\pi}{60} \approx 0.764$
The probability is $\frac{45.86}{60} \approx 0.764$ .	1 point	
<b>Total:</b>	<b>7 points</b>	

*Note:*

*The points marked by \* are also due if the candidate calculates the acute angles of the triangle:  $\alpha \approx 28.07^\circ$ ,  $\beta \approx 61.93^\circ$  (1 point) and also the combined area of the sectors: the area of the sector at vertex A, central angle  $\alpha$ , is about 2.205 cm<sup>2</sup>, the area of the sector at vertex B, central angle  $\beta$ , is about 4.864 cm<sup>2</sup>, the area of the quarter of a circle at vertex C is about 7.069 cm<sup>2</sup> (1 point). The area of the dark part of the triangle is  $(2.205 + 4.864 + 7.069 \approx) 14.14$  cm<sup>2</sup> (1 point).*

<b>6. a)</b>		
5 litres = 5000 cm <sup>3</sup>	1 point	
$V = r^2 \pi m = 15r^2 \pi = 5000$	1 point	
The radius of the base circle of the pot is $r = \sqrt{\frac{5000}{15\pi}} \approx 10.3$ cm.	1 point	
<b>Total:</b>	<b>3 points</b>	

<b>6. b)</b>		
$V = r^2 \pi m = 5000$ , so $m = \frac{5000}{r^2 \pi}$ .	1 point	
(The surface area of the open top cylinder is to be minimised.) $A = r^2 \pi + 2r\pi m = r^2 \pi + 2r\pi \cdot \frac{5000}{r^2 \pi} = r^2 \pi + \frac{10\,000}{r}$	2 points	
The derivative of the function $f(r) = r^2 \pi + \frac{10\,000}{r}$ that is defined over the set of positive real numbers is $f'(r) = 2r\pi - \frac{10\,000}{r^2}$ .	1 point*	
(Function $f$ may assume an extreme wherever its derivative is 0.) $f'(r) = 0$ , so $r = \sqrt[3]{\frac{5000}{\pi}} \approx 11.7$ cm.	2 points*	
$r < \sqrt[3]{\frac{5000}{\pi}}$ makes $f'(r) < 0$ , $r > \sqrt[3]{\frac{5000}{\pi}}$ makes $f'(r) > 0$ , so the function $f$ has a minimum here.	1 point*	$f''(r) = 2\pi + \frac{20\,000}{r^3}$ $f''\left(\sqrt[3]{\frac{5000}{\pi}}\right) > 0$
The minimal amount of enamel is needed when the radius of the base circle of the pot is 11.7 cm.	1 point	
<b>Total:</b>	<b>8 points</b>	

Notes: 1. In this case  $m = r$ , the amount of enamel needed is about 1285 cm<sup>2</sup>.

2. The 4 points marked by \* may also be given for the following reasoning:

$A(r) = r^2 \pi + \frac{10\,000}{r} = r^2 \pi + \frac{5000}{r} + \frac{5000}{r}$ .	1 point	
According to the inequality between the arithmetic and geometric means: $A(r) \geq 3 \cdot \sqrt[3]{r^2 \pi \cdot \frac{5000}{r} \cdot \frac{5000}{r}} = 3 \cdot \sqrt[3]{25\,000\,000 \pi}$ .	2 points	
Equation may occur when $r^2 \pi = \frac{5000}{r}$ , which gives $r = \sqrt[3]{\frac{5000}{\pi}}$ ( $\approx 11.7$ ).	1 point	

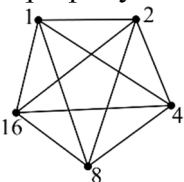
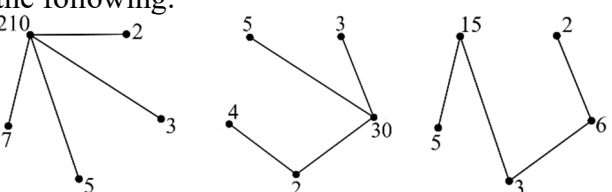
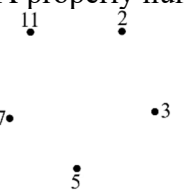
<b>6. c)</b>		
The probability that the selected pot is not damaged is $1 - p$ .	1 point	
The probability that none of the 20 pots are damaged is $(1 - p)^{20}$ .	1 point	
According to the question: $(1 - p)^{20} \geq 0.8$ ,	1 point	
(as $1 - p \geq 0$ ) this means $p \leq 1 - \sqrt[20]{0.8}$ .	1 point	
The maximum value of $p$ is therefore 0.011.	1 point	
<b>Total:</b>	<b>5 points</b>	

<b>7. a)</b>		
$35\,700 = 2^2 \cdot 3 \cdot 5^2 \cdot 7 \cdot 17$	1 point	
Two numbers are co-primes if they do not have a common prime factor. The five different prime numbers (along with their respective exponents) above must, therefore, be divided into two groups.	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
On selecting primes for the first number, a decision must be made for each of the five primes whether they will be selected or not. This gives $2^5 = 32$ different options. (If none of the primes are selected then the value of the number is 1.)	2 points	<i>For example, let 17 be a factor in one of the numbers. It is now enough to select some other primes to go with it (the other number will either be the product of the primes not selected or 1).</i>
However, each pair of numbers will be obtained twice this way (as the groups are interchangeable), so there are only $32 : 2 = 16$ pairs.	1 point	<i>There are <math>2^4 = 16</math> different options to decide about the other four primes (along with their exponents).</i>
<b>Total:</b>	<b>5 points</b>	

*Note: There are 16 such pairs: (1, 35 700), (3, 11 900), (4, 8925), (7, 5100), (12, 2975), (17, 2100), (21, 1700), (25, 1428), (28, 1275), (51, 700), (68, 525), (75, 476), (84, 425), (100, 357), (119, 300), (175, 204).*

<b>7. b) Solution 1</b>		
In any subset $H'$ of the set $H$ the product of the elements will be divisible by 9 if either $9 \in H'$ , or $9 \notin H'$ and $\{3; 6\} \subseteq H'$ .	1 point	<i>This point is also due if the correct reasoning is reflected only by the solution.</i>
If $9 \in H'$ then any one of the other 9 elements may or may not be selected into the subset. There are $2^9 = 512$ such subsets.	1 point	
If $9 \notin H'$ , then $3 \in H'$ and $6 \in H'$ , and the other 7 elements of $H$ may or may not be selected into $H'$ . There are $2^7 = 128$ such subsets.	2 points	
There are a total $(512 + 128 =) 640$ such subsets of $H$ .	1 point	
<b>Total:</b>	<b>5 points</b>	

<b>7. b) Solution 2</b>		
Examine the complement. In any subset $H'$ of the set $H$ the product of the elements will not be divisible by 9 if $9 \notin H'$ and at least one of the numbers 3 and 6 is also not an element of $H'$ .	1 point	
If $9 \notin H'$ and $3 \notin H'$ then any of the other 8 elements of $H$ may or may not be selected to be an element of $H'$ . There are $2^8 = 256$ such subsets.	1 point	<i>The set <math>H</math> has <math>2^7 = 128</math> subsets such that <math>9 \notin H'</math>, <math>3 \notin H'</math> and <math>6 \in H'</math>.</i>
Similarly, there are also 256 subsets of $H$ that does not contain the numbers 9 and 6.	1 point	<i>There are an equal number of subsets such that <math>9 \notin H'</math>, <math>3 \in H'</math> and <math>6 \notin H'</math>, and also such that <math>9 \notin H'</math>, <math>3 \notin H'</math> and <math>6 \notin H'</math>.</i>
However, the subsets that contain neither 9, nor 6, nor 3 have been counted twice. There are $2^7 = 128$ such subsets, so the total number of possible subsets is $(256 + 256 - 128 =)$ 384.	1 point	<i>The total number of “wrong” subsets is <math>3 \cdot 2^7 = 284</math>.</i>
As the set $H$ has a total of $2^{10} = 1024$ subsets the final number of proper subsets is $(1024 - 384=)$ 640.	1 point	
<b>Total:</b>	<b>5 points</b>	

<b>7. c)</b>		
A properly numbered complete graph, for example: 	2 points	
A properly numbered tree graph, for example one of the following: 	2 points	
A properly numbered empty graph, for example: 	2 points	
<b>Total:</b>	<b>6 points</b>	

Notes: 1. Award 1 point for any case in which the candidate writes appropriate numbers next to each point but makes a mistake in drawing the edges.  
 2. Award no points at all if the candidate draws the complete graph and a tree graph without the associated numbers.

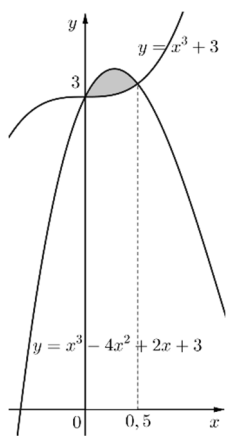
<b>8. a) Solution 1</b>		
(According to the text $v_0 = 18$ m/s and $x = 20$ m.) $v(20) = \sqrt{18^2 - 15 \cdot 20} =$	2 points	
$= \sqrt{24} \approx 4.9$ (m/s) $> 0$ .	1 point	
After covering 20 metres the speed of the car is still about 4.9 m/s ( $\approx 16.7$ km/h) and so it will not stop.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>8. a) Solution 2</b>		
(According to the text $v_0 = 18$ m/s.) $v(x) = 0$ m/s when $18^2 - 15x = 0$ .	2 points	
In this case $x = \frac{18^2}{15} = 21.6$ (m),	1 point	
so the braking distance is greater than 20 metres. It will not stop.	1 point	
<b>Total:</b>	<b>4 points</b>	

<b>8. b)</b>		
(When the car stops $x = 40$ m.) $v(40) = 0$ m/s when $v_0^2 - 15 \cdot 40 = 0$ .	2 points	
When the driver started braking the speed of the car was $v_0 \approx 24.5$ m/s ( $\approx 88$ km/h).	1 point	
<b>Total:</b>	<b>3 points</b>	

<b>8. c)</b>		
The distance covered during the reaction time of the driver is $15 \cdot 0.8 = 12$ (m).	1 point	
The initial velocity is $v_0 = 15$ m/s. $v(x) = 0$ m/s when $15^2 - 15x = 0$ , in this case the braking distance is $x = 15$ metres.	2 points	
On a dry road the total stopping distance is $15 + 12 = 27$ metres.	1 point	
Let the speed of the car on a wet, icy road be $v$ (m/s). The distance covered during the reaction time of the driver is $v \cdot 0.8$ (m).	1 point	
$v^2 - 3x = 0$ , so the braking distance is $x = \frac{v^2}{3}$ (m).	1 point	
$\frac{v^2}{3} + 0.8v = 27$ , so $\frac{v^2}{3} + 0.8v - 27 = 0$ .	1 point	
The positive solution of this equation is $v \approx 7.88$ ,	1 point	
which means the car has to travel at about 7.88 m/s ( $\approx 28.4$ km/h) on a wet, icy road to produce the same stopping distance as that of a car travelling on a dry road at a speed of 15 m/s (54 km/h).	1 point	
<b>Total:</b>	<b>9 points</b>	

<b>9. a)</b>		
$f(0) = c$ , so $c = 1$ , and	1 point	
$f(1) = 0$ so $a + b + 2 = 0$ .	1 point	
$f'(x) = 3x^2 + 2ax + b$	1 point	
$f''(x) = 6x + 2a$	1 point	
$f'(2) = 12 + 4a + b$ and $f''(1) = 6 + 2a$	1 point	
According to the condition $12 + 4a + b = 6 + 2a$ ,	1 point	
$2a + b + 6 = 0$ .	1 point	
The solution of the equation system $\begin{cases} a + b + 2 = 0 \\ 2a + b + 6 = 0 \end{cases}$ is $a = -4$ , $b = 2$ .	2 points	
Check: The function $f: \mathbf{R} \rightarrow \mathbf{R}$ , $f(x) = x^3 - 4x^2 + 2x + 1$ meets all conditions: $f(0) = 1$ , $f(1) = 1 - 4 + 2 + 1 = 0$ , $f'(2) = f''(1) (= -2)$ .	1 point	
<b>Total:</b>	<b>10 points</b>	

<b>9. b)</b>		
Rearranging the equation $x^3 - 4x^2 + 2x + 3 = x^3 + 3$ yields $-4x^2 + 2x = 0$ .	1 point	
This equation has, in fact, two solutions: 0 and 0.5. (The points of intersection are (0; 3) and (0.5; 3.125).)	1 point	
The area of the region will be calculated as $\left  \int_0^{0.5} (x^3 - 4x^2 + 2x + 3 - (x^3 + 3)) dx \right $ .	1 point	$\int_0^{0.5} (x^3 - 4x^2 + 2x + 3) dx -$ $-\int_0^{0.5} (x^3 + 3) dx$
$\left  \int_0^{0.5} (-4x^2 + 2x) dx \right  = \left  \left[ -\frac{4x^3}{3} + x^2 \right]_0^{0.5} \right  = \left  -\frac{1}{6} + \frac{1}{4} - 0 \right  =$	2 points	Using the calculated values of the integrals:
$= \frac{1}{12}$	1 point	$\frac{307}{192} - \frac{97}{64} = \frac{1}{12}$ .
<b>Total:</b>	<b>6 points</b>	